

Intermediate left-right gauge symmetry, unification of couplings and fermion masses in supersymmetric $SO(10) \times S_4$

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If left-right gauge theory, $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C (g_L = g_R) (\equiv G_{2213})$, occurs as an intermediate symmetry in a grand unified theory then, apart from other advantages, it is possible to obtain the seesaw scale necessary to understand small neutrino masses with Majorana coupling of order unity. Barring threshold or non-renormalizable gravitational effects at the GUT scale, or the assumed presence of additional light scalar particles of unprescribed origin, all other attempts to achieve manifest one-loop gauge coupling unification in SUSY $SO(10)$ with such intermediate symmetry have not been successful so far. Attributing this failure to lack of flavor symmetry in the grand unified theory, we show how the spontaneous symmetry breaking of $SO(10) \times S_4$ leads to such left-right intermediate breaking scale extending over the range $M_R \simeq 5 \times 10^9$ GeV to 10^{15} GeV. All the charged fermion masses are fitted at the intermediate see-saw scale, $M_N \simeq M_R \simeq 4 \times 10^{13}$ GeV which is obtained with Majorana coupling $f_0 \simeq 1$. Using type I seesaw and a constrained parametrisation in which CP-violation originates only from the quark sector, in addition to other predictions made in the neutrino sector, the reactor mixing angle is found to be $\theta_{13} \simeq 3^\circ - 5^\circ$ which is in the range accessible to ongoing experiments. The leptonic Dirac phase turns out to be $\delta \simeq 2.9 - 3.1$ radians with the predicted values of Jarlskog invariant $J_{CP} \simeq 2.95 \times 10^{-5} - 10^{-3}$.

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I. INTRODUCTION

$SO(10)$ grand unified theory [1] has a number of attractive features which have resulted in recent surge of investigations including applications to fermion masses and mixings [2]. It unifies all fermions of one generation plus the right-handed neutrino into one spinorial representation **16**. With D-parity as an element of gauge transformation [3], it naturally restores left-right and CP symmetries at the GUT scale and thus, it can provide a spontaneous origin of P and CP-violations [4]. It embodies quark-lepton unification with high predictive power in the fermionic sector [5, 6]. Through its Higgs representations **10** and **$\overline{126}$** or **$\overline{16}$** , it has the potentialities for intermediate $SU(2)_R \times U(1)_{B-L}$ breaking, generation of small Majorana neutrino masses through type-I, type-II, and type III see-saw mechanisms [7, 8, 9], explanation of large neutrino mixings through type-II see-saw dominance, accounting for dominant contributions to charged fermion masses through the **10**-representation and providing the desired corrections to them through the weak-doublets in **$\overline{126}$** and **120** [10, 11, 12, 13, 14, 15, 16]. With natural R-parity conservation, in addition to ensuring proton stability [17], it predicts the lightest supersymmetric particle as a stable dark matter candidate.

All fermion masses and mixings, including very small masses and large mixings in the neutrino sector, have been shown to fit reasonably well if the right-handed neutrino mass scale is in the range 10^{13} GeV to 10^{14} GeV [12, 16]. Further, thermal leptogenesis explaining origin of matter through baryogenesis can be implemented if the right handed neutrino masses are in intermediate range [18, 19]. Therefore, it would be interesting to obtain the right-handed mass scale near the intermediate breaking of $SU(2)_L \times SU(2)_R \times SU(4)_C (\equiv G_{224}) \subset SO(10)$ or any of its subgroups like G_{2213} . However, the mass spectra analysis in the minimal SUSY $SO(10)$ model [20] with the Higgs representations $210 \oplus 126 \oplus \overline{126} \oplus 10$ rules out any possibility of intermediate gauge symmetry by predicting additional light scalars which are found to disrupt gauge coupling unification [21, 22, 23]. It has been also noted that, even if these additional light scalars are made naturally superheavy using a non-minimal Higgs representation, leaving only the minimal light Higgs spectrum necessary to implement spontaneous symmetry breaking in the presence of supersymmetry and R-parity conservation, there exists no intermediate scale through manifest one-loop unification of gauge couplings.

Recently attempts have been made to obtain desired see-saw scales and improved fits to the fermion masses using different types of mechanisms or by invoking GUTs in higher dimensions [15, 24]. In addition, extensions of gauge symmetries by non-abelian flavor groups like S_3, S_4 , and A_4 have resulted in interesting consequences [6, 25, 26, 27, 28, 29] including high scale unification of quark and lepton mixings [6].

Since SUSY GUTs with a left-right intermediate gauge symmetry has many attractive features over super-grand desert models, in this paper we discuss a novel procedure of constructing such a model through an extension of the left-right gauge symmetry to $G_{2213} \times S_4$ and the corresponding extension of the GUT symmetry to $SO(10) \times S_4$ to encompass supersymmetric flavor unification, grand unification, and R-parity conservation [30]. We find that when

such a flavor symmetry is included, manifestly successful one-loop gauge coupling unification occurs with the three gauge couplings of the left-right gauge theory attaining convergence to the unification coupling at the GUT scale. The intermediate scale is predicted over a wide range with $M_R \approx 5 \times 10^9$ GeV to 10^{15} GeV. The desired Higgs scalars necessary for the gauge coupling unification are found to be consistent with the mass spectra analysis in the $SO(10) \times S_4$ model. Although the contribution of three fermion generations cancel out from one-loop unification, flavor symmetry requires enlarged Higgs spectrum which modifies the beta function coefficients and the evolution of the gauge couplings leading to successful unification in the presence of $G_{2213} \times S_4$ intermediate symmetry. In the second part of the paper, we fit the renormalization group (RG) extrapolated data on fermion masses, mixings, and phases at the intermediate see-saw scale $M_N \simeq M_R \simeq 10^{13}$ GeV and obtain successful predictions in the neutrino sector.

This paper is organized in the following manner. In Sec.2 we briefly review problems associated with realization of an intermediate scale in SUSY $SO(10)$. In Sec. III we discuss how the desired intermediate scale is achieved via left-right gauge and S_4 flavor symmetries. Symmetry breaking of $SO(10) \times S_4$ is discussed in Sec.IV. In Sec.V we show how the desired light particle spectrum is obtained from the $SO(10) \times S_4$ theory. Fits to the fermion masses and mixings and model predictions in the neutrino sector are carried out in Sec.VI. A brief summary of investigations made and conclusions obtained are stated in Sec.VII.

II. DIFFICULTIES IN R-PARITY CONSERVING LEFT-RIGHT INTERMEDIATE GAUGE SYMMETRY

In this section we discuss briefly the problems associated with obtaining a $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C (g_L = g_R) (\equiv G_{2213})$ intermediate gauge symmetry in R-parity conserving supersymmetric $SO(10)$ grand unified theory through one-loop unification of gauge couplings.

For this purpose, we consider two-step breaking of $SO(10)$ to the minimal supersymmetric standard model(MSSM) through G_{2213} intermediate gauge symmetry in the so called minimal grand unified theory,

$$\begin{aligned} SO(10) &\xrightarrow[M_U]{210} G_{2213} \\ &\xrightarrow[M_R]{126 \oplus \overline{126}} G_{213} \xrightarrow[M_Z]{10} U(1)_{em} \times SU(3)_C. \end{aligned} \quad (1)$$

The G_{224} sub-multiplet $(1, 1, 15)$ in $\Phi(210)$ contains a G_{2213} singlet which is even under D-parity [3]. When this component acquires VEV, $SO(10) \rightarrow G_{2213}$ with unbroken left-right discrete symmetry.

Unlike the D-parity breaking case where the intermediate left-right gauge group has four different coupling constants, in the present case G_{2213} has only three gauge couplings, $g_{2L} = g_{2R}, g_{3C}$, and g_{BL} for $\mu \geq M_R$. In the second step, the right-handed triplet component in $\overline{126}$ acquires VEV to break $G_{2213} \rightarrow G_{213}$ while generating heavy right-handed Majorana neutrino mass. In the process of spontaneous electro-weak symmetry breaking driven by weak bi-doublet in **10**, small left-handed neutrino masses are generated through Type I and Type II see-saw mechanisms.

To discuss gauge coupling unification we use the following three RGEs from M_Z to M_U [31, 32, 33],

$$\begin{aligned} \frac{1}{\alpha_Y(M_Z)} &= \frac{1}{\alpha_G} + \frac{a_Y}{2\pi} \ln \frac{M_R}{M_Z} + \frac{1}{10\pi} (3a'_{2L} + 2a'_{BL}) \ln \frac{M_U}{M_R}, \\ \frac{1}{\alpha_{2L}(M_Z)} &= \frac{1}{\alpha_G} + \frac{a_{2L}}{2\pi} \ln \frac{M_R}{M_Z} + \frac{a'_{2L}}{2\pi} \ln \frac{M_U}{M_R}, \\ \frac{1}{\alpha_{3C}(M_Z)} &= \frac{1}{\alpha_G} + \frac{a_{3C}}{2\pi} \ln \frac{M_R}{M_Z} + \frac{a'_{3C}}{2\pi} \ln \frac{M_U}{M_R}. \end{aligned} \quad (2)$$

where α_G is the GUT fine-structure constant and the beta function coefficients a_i and a'_i are determined by the particle spectrum in the ranges from M_Z to M_R , and from M_R to M_U , respectively. Adopting the standard procedure we obtain the following two equations [33]

$$L_\theta \equiv \frac{2\pi}{\alpha(M_Z)} \left(1 - \frac{8 \sin^2 \theta_W(M_Z)}{3} \right) = A \ln \frac{M_U}{M_Z} + B \ln \frac{M_R}{M_Z}, \quad (3)$$

$$L_S \equiv \frac{2\pi}{\alpha(M_Z)} \left(1 - \frac{8\alpha(M_Z)}{3\alpha_{3C}(M_Z)} \right) = A' \ln \frac{M_U}{M_Z} + B' \ln \frac{M_R}{M_Z}. \quad (4)$$

where

$$\begin{aligned} A &= \frac{2}{3}(a'_{BL} - a'_{2L}), \quad B = \frac{5}{3}(a_Y - a_{2L}) - \frac{2}{3}(a'_{BL} - a'_{2L}), \\ A' &= 2a'_{2L} + \frac{2}{3}a'_{BL} - \frac{8}{3}a'_{3C}, \quad B' = \frac{5}{3}a_Y + a_{2L} - \frac{8}{3}a_{3C} - (2a'_{2L} + \frac{2}{3}a'_{BL} - \frac{8}{3}a'_{3C}). \end{aligned} \quad (5)$$

From eq.(3) and eq.(4), the analytic expressions for M_U and M_R immediately follow,

$$\ln \frac{M_U}{M_Z} = \frac{1}{(AB' - A'B)} (B'L_\theta - BL_S), \quad (6)$$

$$\ln \frac{M_R}{M_Z} = \frac{1}{(AB' - A'B)} (AL_S - A'L_\theta). \quad (7)$$

Using PDG values, $\alpha(M_Z) = 127.9$, $\sin^2\theta_W(M_Z) = 0.2312$, and $\alpha_{3C}(M_Z) = 0.1187$ [34], we obtain

$$L_S = 662.736, \quad L_\theta = 308.305. \quad (8)$$

Ignoring the contributions from additional lighter scalar multiplets emerging from mass spectra analysis which have been discussed later, the minimal Higgs content necessary to break SUSY $SO(10)$ through G_{2213} to the low energy group and the associated beta function coefficients are,

$\mu = M_Z - M_R$:

$$H^u(2, 1, 1) \oplus H^d(2, -1, 1) \subset G_{213}, \quad (9)$$

$$a_Y = 33/5, a_{2L} = 1, a_{3C} = -3, \quad (10)$$

$\mu = M_R - M_U$:

$$H^\phi(2, 2, 0, 1), \Delta_L(3, 1, -2, 1) \oplus \Delta_R(1, 3, -2, 1) \oplus \overline{\Delta}_L(3, 1, 2, 1) \oplus \overline{\Delta}_R(3, 1, 2, 1), \quad (11)$$

under G_{2213} with

$$\begin{aligned} a'_{BL} &= 24, \quad a'_{2L} = a'_{2R} = 5, \\ a'_{3C} &= a_{3C} = -3. \end{aligned} \quad (12)$$

Thus, the one-loop coefficients give,

$$A = 38/3, \quad B = -10/3, \quad A' = 34, \quad B' = -14, \quad AB' - A'B = -64, \quad (13)$$

leading to the solutions [35],

$$M_R \simeq 10^{16} \text{ GeV}, \quad M_U = 2 \times 10^{16} \text{ GeV}. \quad (14)$$

The mass spectra analysis in the minimal SUSY $SO(10)$ with $210 \oplus 126 \oplus \overline{126} \oplus 10$ predicts that there are additional scalar components of 210 having intermediate scale mass with the following G_{2213} quantum numbers [23],

$$(3, 1, -2/3, 3), (3, 1, 2/3, \overline{3}), (1, 3, -2/3, 3), (1, 3, 2/3, \overline{3}) \quad (15)$$

Not only these states prevent any value of intermediate scale below M_U , as explicitly noted in Ref. [23], but, as noted in Ref. [35], their presence at scales substantially lower than 2×10^{16} GeV also spoils perturbative renormalization of gauge couplings.

It has been pointed out that G_{2213} intermediate scale can still be obtained in non-minimal $SO(10)$ by threshold effects or by the presence of non-renormalizable dim.5 operators in the $SO(10)$ Lagrangian [35, 36]. These might arise if, in addition to **210**, the theory contains a Higgs representation **54**. The implementation of the gauge coupling unification has been found to be possible by threshold corrections or by gravitational corrections in non-renormalizable SUSY $SO(10)$ only if the additional light scalars given in eq. (15) are made superheavy which could be realized due to the added presence of **54** [35].

However, the more attractive and popular unification scheme being through manifestly one-loop evolution of gauge couplings in a renormalizable grand unified theory where coupling constants from lower scale evolve to converge to the unification coupling at the GUT scale, in the next two sections we show how such a unification is achieved when the flavor symmetry S_4 is combined with $SO(10)$ as well as the R-parity and parity conserving supersymmetric left-right gauge theory G_{2213} at the intermediate scale.

III. INTERMEDIATE LEFT-RIGHT GAUGE SYMMETRY WITH S_4 FLAVOR SYMMETRY

In this section we show that in the presence of flavor symmetry and left-right gauge symmetry, the extrapolation of standard model gauge couplings through $G_{2213} \times S_4$ intermediate symmetry naturally leads to successful unification of gauge couplings at 2×10^{16} GeV. In the next section we show how the minimal particle content necessary for this intermediate symmetry follows from mass spectrum analysis of supersymmetric $SO(10) \times S_4$.

It is well known that the minimal particle content of the standard model (SM) alone in non-supersymmetric theory does not allow its gauge couplings to unify at any higher scale. However, when the SM emerges from nonSUSY left-right symmetric G_{2213} at the intermediate scale, profound unification of gauge couplings occurs at the GUT scale [37]. Since the non-SUSY theories are well known for their generic gauge hierarchy problem, when supersymmetry is combined with the SM to solve this problem, the enlarged particle spectrum naturally infused into the MSSM, achieves gauge coupling unification at the SUSY GUT scale without any intermediate scale. On the other hand, if an intermediate symmetry with extended gauge group like G_{2213} is introduced, manifest one-loop gauge coupling unification is spoiled in SUSY GUTs.

Earlier, without ascribing any connection with flavor symmetry, several authors have noted that when the particle spectrum at lower scales is further extended beyond the minimal spectrum, manifest one-loop gauge coupling unification occurs in the presence of G_{2213} intermediate symmetry with or without parity or R-parity [35, 38].

These observations lead us to suggest that the present failure to achieve manifest one-loop gauge coupling unification with intermediate left-right gauge symmetry may be hinting at its extension to include a family symmetry with corresponding extension in the particle spectrum. We find that this new symmetry to be appended to G_{2213} and $SO(10)$ could be the well known flavor symmetry S_4 [6, 27, 28].

This conclusion can be inferred by also looking into the structure of the RGEs in eq.(3) and eq.(4). It is clear that if the coefficients B and B' are negligible compared to A and A' , then the solutions for the mass scales would be insensitive to the values of M_R . Then values of M_R substantially lower than the SUSY GUT scale could be tolerated by the RG constraints. We find that this possibility can be realised within $SO(10) \times S_4$.

In the enlarged particle spectrum, along with the two sets of Higgs triplets of the minimal scenario given in eq.(12), the presence of S_4 symmetry needs six bi-doublets instead of only one [28]. In addition, other scalar multiplets belonging to **1**, **2**, or **3** of S_4 having nontrivial transformation property under G_{2213} are also found to be essential. These latter Higgs particles turn out to be a triplet **3** of S_4 and each member of the triplet transforms as a color octet. The fermions of three generations are taken as **3'** of S_4 .

Thus, keeping the MSSM particle spectrum from M_Z to M_R unaltered, the enlarged Higgs spectrum at the intermediate scale consistent with $G_{2213} \times S_4$ symmetry is:

$$\mu = M_R - M_U:$$

$$\begin{aligned} \Delta_L(3, 1, -2, 1) \oplus \Delta_R(1, 3, -2, 1) \oplus \overline{\Delta}_L(3, 1, 2, 1) \oplus \overline{\Delta}_R(3, 1, 2, 1), \\ 6(2, 2, 0, 1), 3(1, 1, 0, 8), \end{aligned} \quad (16)$$

where **6** = **3** + **2** + **1**, and **3**, **2** and **1** are triplet, doublet, and singlet, respectively, under S_4 . These Higgs scalars modify the beta-function coefficients to

$$a'_{BL} = 24, a'_{2L} = a'_{2R} = 10, a'_{3C} = 6. \quad (17)$$

Noting that the value of $a'_{BL} = 24$ which is the same as in eq.(12), determines the perturbative constraint on the lowest allowed value of M_R [35], we have with the above Higgs spectrum,

$$A = \frac{28}{3}, A' = 20, B = B' = 0. \quad (18)$$

It is interesting to note that the particular combination of $G_{2213} \times S_4$ given in eq.(16) leads to exactly vanishing values of B and B' . The fact that now B and B' both vanish with such Higgs content ensures the possibility of an intermediate scale over wide range of values. But the perturbative lower bound on M_R being determined by $a'_{BL} = 24$ due to the appearance of a Landau pole, now the popular one loop unification of gauge couplings is expected to materialise for values of the intermediate scale satisfying this bound [35].

As eq.(6) and eq.(7) are no longer valid with $B = B' = 0$, we solve for the mass scales numerically using eqs.(2), (10), and (17). We find all values of the left-right symmetry breaking scale M_R are permitted over a wide range,

$$5 \times 10^9 \text{ GeV} \leq M_R \leq 10^{16} \text{ GeV}. \quad (19)$$

but having almost the same value of unification scale $M_U = 2 \times 10^{16} \text{ GeV}$ for all solutions. Two examples of such solutions for $M_R = 10^{13} \text{ GeV}$ and $M_R = 5 \times 10^9 \text{ GeV}$ are shown in Fig. 1 and Fig. 2, respectively. where manifest one-loop unification with the three gauge couplings of G_{2213} converging at the GUT scale, $M_U = 2 \times 10^{16} \text{ GeV}$, is evident.

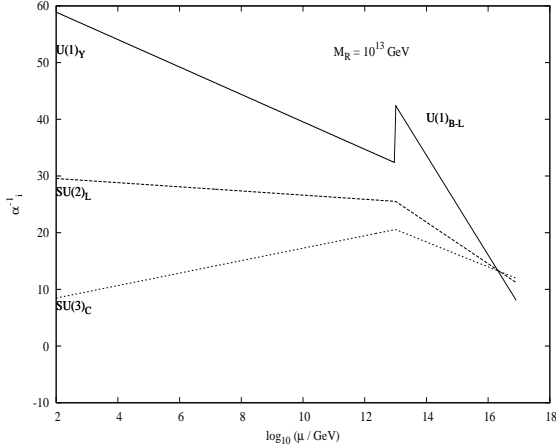
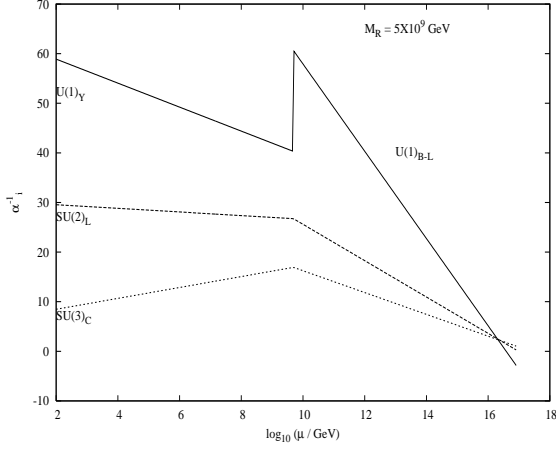


FIG. 1: Evolution of gauge couplings showing variation of inverse fine-structure constants $\alpha_i^{-1}(\mu)$ as a function of the mass scale μ with $G_{2213} \times S_4$ intermediate gauge symmetry breaking at $M_R = 10^{13} \text{ GeV}$ in SUSY $SO(10) \times S_4$ model. The top solid line represents $\alpha_Y^{-1}(\mu)$ for $\mu = M_Z - M_R$ and $\alpha_{B-L}^{-1}(\mu)$ for $\mu = M_R - M_U$. The middle and the bottom lines represent $\alpha_{2L}^{-1}(\mu)$ and $\alpha_{3C}^{-1}(\mu)$, respectively, throughout the range of μ .

For $M_R \simeq 10^{15} \text{ GeV}$, the GUT fine structure constant has the value $\alpha_G \simeq 1/24.5$ which increases as M_R approaches lower values, reaching $\alpha_G \simeq 1/12$ and $\alpha_G \simeq 1/2$ at $M_R = 10^{13} \text{ GeV}$ and $M_R = 5 \times 10^9 \text{ GeV}$, respectively. This phenomenon is due to the appearance of Landau pole near $M_U = 2 \times 10^{16} \text{ GeV}$ in the gauge coupling of $U(1)_{B-L}$ when $M_R \leq 10^9 \text{ GeV}$. Now that all the three gauge couplings of G_{2213} are unified at the GUT scale, α_G would approach ∞ for the same value of $M_R \leq 10^9 \text{ GeV}$. The basic reason is that $a'_{B-L} = 24$ has remained the same as the minimal model inspite of new contributions from $G_{2213} \times S_4$ multiplets.

The $SU(2)_R$ gauge coupling at $\mu \simeq M_R$ is nearly $g_L = g_R = g \simeq 0.7$. With the $SU(2)_R$ gauge boson mass scale $M_R \simeq gv_R$, the vacuum expectation value of the Higgs triplet field is also allowed in the similar range with

FIG. 2: Same as Fig. 1 but for $M_R = 5 \times 10^9$ GeV.TABLE I: Particle content of the model and their transformation properties under $S_4 \times SO(10)$

Fermions	Higgs Bosons						
$\Psi_i, (i = 1, 2, 3)$	S	Φ	$\mathbf{A}_{1,2,3}$	$\overline{\Sigma}_0 \oplus \Sigma_0$	\mathbf{H}_0	$\mathbf{H}_{1,2}$	$\mathbf{H}_{3,4,5}$
$3' \times 16$	1×54	1×210	3×45	$1 \times \overline{126} \oplus 126$	1×10	2×10	3×10

$< \overline{\Delta}_R^0 > = v_R \simeq 5 \times 10^9$ GeV to 10^{16} GeV. Ignoring the constraint from the neutrino oscillation data which will be discussed in Sec.6, the right-handed neutrino mass $M_N = f_0 v_R$ for Majorana coupling $f_0 \simeq 1$ is then allowed to vary over similar range without the necessity of any tuning of f_0 .

In the next section we show how the minimal particle content needed for gauge coupling and flavor unification through $G_{2213} \times S_4$ can be easily embedded in $SO(10) \times S_4$. We further show how the F-term flatness condition leads to the desired minimal particle spectrum of the model below the GUT scale while keeping all other components of the $SO(10) \times S_4$ representations superheavy.

IV. SYMMETRY BREAKING OF $SO(10) \times S_4$

We consider S_4 flavor symmetry for three fermion generations and supersymmetric grand unification of three forces of nature through $SO(10) \times S_4$ [27, 28]. Instead of starting from flavor symmetric standard model without SUSY discussed in Ref. [28], we consider flavor symmetry starting at the intermediate scale through $G_{2213} \times S_4$. Such a model leading to MSSM at lower scales originates from intermediate breaking of $SO(10) \times S_4$,

$$SO(10) \times S_4 \rightarrow_{M_U}^{210} G_{2213} \times S_4 \rightarrow_{M_R}^{126 + \overline{126}} G_{213} \rightarrow_{M_W}^{10} U(1)_{em} \times SU(3)_C.$$

To achieve successful gauge coupling unification with the desired intermediate scale, we need the minimal particle content same as in the MSSM from M_Z to M_R . In the presence of $G_{2213} \times S_4$, we need six bi-doublets each of which is embeddable in $H(10)_0, H(10)_{1,2}, H(10)_{3,4,5}$ of $SO(10)$ and these transform as singlet, doublet, and triplet, respectively, under S_4 . The left and the right-handed Higgs triplets needed to maintain supersymmetry and implement spontaneous breaking of G_{2213} at scale M_R are contained in $\Sigma_0(126)$ and $\overline{\Sigma}_0(\overline{126})$ of $SO(10)$. We note that, in addition to the minimal number of doublets and triplets, the gauge coupling unification also needs three $SU(3)_C$ octets transforming as $(1, 1, 0, 8)$ under G_{2213} . Each of them is contained in the G_{224} multiplet $(1, 1, 15)$ which, in turn, is contained in the $A_i(45)$ ($i = 1, 2, 3$) of $SO(10)$ treated as a triplet $\mathbf{3}$ of S_4 . All other Higgs particles are to become superheavy with masses near the GUT scale for successful gauge coupling unification at one-loop level and this can be achieved in the presence of $S(54)$ of $SO(10)$ [24]. The Fermion and Higgs representations are given in Table I.

In order to realize such a spectrum by actual potential minimization in the presence of supersymmetry, we break $SO(10)$ by giving GUT-scale vacuum expectation values to the two D-parity conserving singlets of $\Phi(210)$ and

$S(54)$. We follow the mass spectrum analysis technique for supersymmetric $SO(10)$ grand unification [21, 22, 23]. We assign nearly equal vacuum expectation values to both the singlets with $\langle S \rangle \simeq \langle \Phi \rangle$ such that, effectively, the GUT symmetry breaking to G_{2213} appears as one step process.

Although for the sake of gauge coupling unification alone, it is possible to treat the effective gauge symmetry below M_R as $MSSM \times S_4$ with only one weak bi-doublet having mass near the electro-weak scale, for accommodating all fermion masses and mixings in this model as discussed in the next section, it is necessary to break the flavor symmetry at the intermediate scale.

We assume that G_{2213} gauge symmetry is broken at the scale M_R due to vacuum expectation value of the standard model singlet contained in the right handed triplet $\overline{\Delta}_R(1, 3, 2, 1)$ in $\overline{126}$. After the $G_{2213} \times S_4$ breaking, only two MSSM Higgs doublets are taken to remain light.

The superpotential near the GUT scale can be written as,

$$\begin{aligned} W_H = & \frac{1}{2}m_\Phi\Phi^2 + \frac{1}{2}m_S S^2 + \frac{1}{2}m_A \sum_i A_i^2 + m_\Sigma \Sigma_0 \overline{\Sigma}_0 + \frac{1}{2}m_{H_0} H_0^2 \\ & + \frac{1}{2}m_{H_D} H_D^2 + \frac{1}{2}m_{H_T} H_T^2 + \lambda_0 \Phi^3 + \lambda_1 \Phi \Sigma_0 \overline{\Sigma}_0 + (\lambda_2 \Sigma_0 + \lambda_3 \overline{\Sigma}_0) H_0 \Phi + \lambda_4 \sum_i A_i^2 \Phi \\ & + S(\lambda_5 S^2 + \lambda_6 \sum_i A_i^2 + \lambda_7 \Phi^2 + \lambda_8 \Sigma_0^2 + \lambda_9 \overline{\Sigma}_0^2 + \lambda_{10} H_0^2 + \lambda_{11} H_D^2 + \lambda_{12} H_T^2). \end{aligned} \quad (20)$$

Using vacuum expectation values $\langle S \rangle = v_S, \langle \Phi \rangle = v_\Phi, \langle \Delta_R^\sigma \rangle = \sigma, \langle \overline{\Delta}_R^\sigma \rangle = \overline{\sigma}$, the vanishing F-terms yield [22],

$$m_\Phi v_\Phi + \frac{l_0 v_\Phi^2}{3\sqrt{2}} + \frac{l_1 \sigma \overline{\sigma}}{10\sqrt{2}} - \frac{2l_7 v_\Phi v_S}{\sqrt{15}} = 0, \quad (21)$$

$$m_S v_S + \frac{\sqrt{3}l_5 v_S^2}{2\sqrt{5}} - \frac{l_7 v_\Phi^2}{\sqrt{15}} = 0, \quad (22)$$

$$\left[m_\Sigma + \frac{l_1 v_\Phi}{10\sqrt{2}} \right] \sigma = 0. \quad (23)$$

Due to the vanishing D-term, $\sigma = \overline{\sigma} \equiv v_R$ and the corresponding F-terms for σ or $\overline{\sigma}$ yield the same equation as eq.(23). In the desired hierarchical case, both σ and $\overline{\sigma}$ are much smaller compared to $\langle S \rangle, \langle \Phi \rangle$ leading to the relation between the GUT-scale VEVs and m_Φ ,

$$m_\Phi + \frac{l_0 v_\Phi}{3\sqrt{2}} - \frac{2l_7 v_S}{\sqrt{15}} = 0. \quad (24)$$

Using v_Φ from eq.(24) in eq.(22) gives a quadratic equation for v_S ,

$$p v_S^2 + q v_S - r = 0, \quad (25)$$

where

$$\begin{aligned} p &= \frac{\sqrt{3}l_5}{2\sqrt{5}} - \frac{24l_7^3}{5\sqrt{15}l_0^2}, \\ q &= m_S + \frac{24l_7^2}{5l_0^2} m_\Phi, \\ r &= \frac{18l_7}{\sqrt{15}l_0^2} m_\Phi^2. \end{aligned} \quad (26)$$

In the next section we discuss the emergence of mass spectra necessary to keep only the desired minimal number of Higgs particles light while making others superheavy.

V. LIGHT AND HEAVY PARTICLE STATES FROM MASS SPECTRA

In this section we discuss the emerging mass spectra from the spontaneously broken flavor symmetric GUT while making provisions for would be Goldstone bosons and the light scalars necessary for gauge coupling unification. In contrast to the minimal model without flavor symmetry where unwanted light scalar degrees of freedom are found to spoil gauge coupling unification [23], in the present case, due to the presence of the scalar multiplet **54** in the Higgs superpotential, it is possible to lift those masses to the GUT scale.

A. Goldstone Bosons

In the process of spontaneous symmetry breaking of $SO(10) \rightarrow G_{2213}$ through $\langle S \rangle$ and $\langle \Phi \rangle$, 30 gauge bosons would acquire GUT-scale mass by absorbing the corresponding mass-less scalars. Under G_{2213} these superheavy gauge bosons have the quantum numbers $(2, 2, 2/3, 3) + (2, 2, -2/3, \bar{3}) + (1, 1, 2/3, 3) + (1, 1, -2/3, \bar{3})$. Whereas the first two sets of states are contained in both $(2, 2, 6) \subset G_{224} \subset 54$ and $(2, 2, \bar{10}) \subset G_{224} \subset 210$ of $SO(10)$, the next two sets of states are contained in $(1, 1, 15) \subset G_{224} \subset 210$ or 45 of $SO(10)$. Using the superpotential in eq.(20) and the vacuum expectation values, we show how the desired Goldstone bosons are obtained.

A.1. $(1, 1, 3, 2/3) + (c.c)$ as Goldstone Bosons

Noting that $\sigma = \bar{\sigma} \equiv v_R \ll v_S \sim v_\Phi \sim M_U$, it turns out that these unmixed states in the leading approximation have masses,

$$m_{G1} = m_\Phi + \frac{l_0 v_\Phi}{3\sqrt{2}} - \frac{2l_7 v_S}{\sqrt{15}}. \quad (27)$$

Using eq.(24) it is immediately recognised that these are naturally the light pseudo Goldstone bosons to be absorbed by the mass-less vector bosons to make them superheavy. It can be easily checked that other unmixed states having the same quantum numbers in $A_i (i = 1, 2, 3)$ have degenerate superheavy masses,

$$m_A + \frac{\sqrt{2}l_4 v_\Phi}{\sqrt{3}} - \frac{2l_6 v_S}{\sqrt{15}}, \quad (28)$$

which are naturally near the GUT scale.

A.2. $(2, 2, 3, 1/3) + (c.c)$ as Goldstone Bosons

Using the basis $[(\mathbf{A}_1)_{(2,2,6)}^{(2,2,1/3,3)}, (\mathbf{A}_2)_{(2,2,6)}^{(2,2,1/3,3)}, (\mathbf{A}_3)_{(2,2,6)}^{(2,2,1/3,3)}, \mathbf{S}_{(2,2,6)}^{(2,2,1/3,3)}, \Phi_{(2,2,6)}^{(2,2,1/3,3)}, \Phi_{(2,2,10)}^{(2,2,1/3,3)}]$ [22] where the superscripts(subscripts) refer to gauge quantum numbers of the Higgs multiplets under $G_{2213}(G_{224})$, there are three unmixed pairs of states in A_1 and A_2 and A_3 with degenerate superheavy masses,

$$m_A + \frac{l_6}{2\sqrt{15}}v_S. \quad (29)$$

The fourth unmixed pair is that of $\Phi_{(2,2,6)}^{(2,2,1/3,3)}$ with superheavy mass,

$$m_\Phi + \frac{7l_7}{4\sqrt{15}}v_S. \quad (30)$$

The remaining two pairs of states, $\mathbf{S}_{(2,2,6)}^{(2,2,1/3,3)} \oplus (c.c.)$ and $\Phi_{(2,2,10)}^{(2,2,1/3,3)} \oplus (c.c.)$, mix through the mass matrix,

$$M_1 = \begin{bmatrix} m_S + \frac{\sqrt{3}l_5 v_S}{2\sqrt{5}} & \frac{l_7 v_\Phi}{2\sqrt{3}} \\ \frac{l_7 v_\Phi}{2\sqrt{3}} & m_\Phi + \frac{l_0 v_\Phi}{3\sqrt{2}} - \frac{\sqrt{3}l_7 v_S}{4\sqrt{5}} \end{bmatrix}. \quad (31)$$

It is clear that one linear combination of these two pairs can be made mass-less by tuning the parameter l_5 such that it supplies the remaining Goldstone modes. The other orthogonal combination acquires mass near the GUT scale.

B. Light Scalars from Mass Spectra for Gauge Coupling Unification

In Sec.3, the unification of gauge couplings with $G_{2213} \times S_4$ intermediate gauge symmetry has been shown to require the usual left- and the right-handed triplets that are contained in $126 \oplus \overline{126}$, six bi-doublets contained in six 10-plets, and a set of three color octets transforming as $(1, 1, 0, 8)$ under G_{2213} . These octets are contained in the G_{224} submultiplet $(1, 1, 15)$ of $A_i(45)$ or $\Phi(210)$ of $SO(10)$. In addition, the G_{224} submultiplet $(1, 1, 20')$ in $S(54)$ also contains the octet component. But we will find it convenient to obtain these three octets from a triplet of $45_i (i = 1, 2, 3) \subset SO(10)$.

At first it is to be noted that the triplets $\Delta_L(3, 1, -2, 1), \Delta_R(1, 3, -2, 1)$ and their conjugates contained $126 \oplus \overline{126}$ acquire degenerate masses,

$$M_R = m_\Sigma + \frac{l_1 v_\Phi}{10\sqrt{2}}. \quad (32)$$

The condition $M_R \ll M_U$ can be ensured by tuning l_1 . At first the six bi-doublets from the six 10-plets are treated to have masses near M_R in the usual fashion by some doublet triplet splitting mechanism or by tuning the parameters l_{10}, l_{11} , and l_{12} while the weak bi-doublets in $126 \oplus \overline{126}$ and 210 are kept heavy at the GUT scale. In the next section we show how five linear combinations of these bi-doublets can be treated to have masses at the M_R -scale while keeping the mass of the remaining linear combination at the electro-weak scale, thus supplying the pair of two MSSM doublets (H^u, H^d) .

Choosing the basis $(A_i)_{1,1,15}^{1,1,0,8}, S_{1,1,20'}^{1,1,0,8}, \Phi_{1,1,15}^{1,1,0,8}$, we find that there are three unmixed states in A_i with masses,

$$m_A = \frac{\sqrt{2}l_4 v_\Phi}{3} - \frac{2l_6 v_S}{\sqrt{15}}. \quad (33)$$

Clearly the advantage of A_i being the members of $\mathbf{3} \subset \mathbf{S}_4$ is that the tuning of the single parameter l_4 makes all the three octets light having masses near the M_R scale which is essential for gauge coupling unification. It is found that the other two states mix through the mass matrix,

$$M_2 = \begin{bmatrix} m_S - \frac{2\sqrt{3}l_5 v_S}{\sqrt{5}} & -\frac{l_7 v_\Phi}{\sqrt{6}} \\ -\frac{l_7 v_\Phi}{\sqrt{6}} & m_\Phi - \frac{l_0 v_\Phi}{3\sqrt{2}} - \frac{2l_7 v_S}{\sqrt{15}} \end{bmatrix}. \quad (34)$$

The eigen values emerging from eq. (34) are at the GUT scale and we do not adopt any further fine-tuning.

We have verified that all the components from G_{224} multiplets, $(1, 3, 15) \oplus (3, 1, 15) \subset \mathbf{210}$ acquire masses near the GUT scale due to the presence of $\mathbf{54}$ in the model and there are no other lighter states which are likely to disrupt gauge coupling unification.

In summary the theory has enough parameter space for the successful implementation of the model with $G_{2213} \times S_4$ intermediate symmetry and gauge coupling unification at the GUT scale via $SO(10) \times S_4$.

VI. FERMION MASSES AND MIXINGS

In this section we address the question of fermion masses and mixings and make predictions in the neutrino sector. For this purpose we assume type-I seesaw dominance and also include a pair of $\mathbf{126}_{1,2} \oplus \overline{\mathbf{126}}_{1,2} \equiv \Sigma_{1,2} \oplus \overline{\Sigma}_{1,2}$ as members of an S_4 doublet but with all their components having GUT scale masses [39]. Even though all the weak bi-doublets $\overline{\Delta}_i^\phi (i = 0, 1, 2)$ contained in the G_{224} submultiplets $(\mathbf{2}, \mathbf{2}, \mathbf{15})$ of $\overline{\Sigma}_i, (i = 0, 1, 2)$ have GUT scale masses, it is necessary to clarify how their induced VEVs contribute to the Dirac masses of all fermions in a manner analogous to minimal SUSY $SO(10)$ [10].

A. Light Weak Bi-doublets and Vacuum Expectation Values

In this subsection we clarify how by keeping only the desired minimal number of particles below the GUT scale, VEV of different weak bi-doublets are made available to generate fermion masses without disrupting gauge coupling unification.

The added S_4 -doublet fields $\Sigma_D \oplus \overline{\Sigma}_D$ will have a new contributions to the superpotential which include,

$$W'_H = m_{\Sigma'} \Sigma_D^T \overline{\Sigma_D} + \lambda'_1 \Phi \Sigma_D^T \overline{\Sigma_D} + (\lambda'_2 \Sigma_D^T + \lambda'_3 \overline{\Sigma_D^T}) H_D \Phi \dots \dots \quad (35)$$

where ellipses denote couplings to other Higgs representations and subscript D stands to indicate a S_4 -doublet. Since $m_{\Sigma'} \simeq M_U$ and we need no fine-tuning of parameters $\lambda'_{1,2,3}$ to maintain gauge hierarchy, it immediately follows that all the components of the S_4 -doublet pair (including the weak bi-doublets and triplets) acquire masses at the GUT scale and they do not upset gauge coupling unification. Denoting the full superpotential as $\mathbf{W} = \mathbf{W}_H + \mathbf{W}'_H$, the \mathbf{F} -term due to Φ now contributes the following terms to the scalar potential,

$$V = (\lambda_2 \Sigma_0 + \lambda_3 \overline{\Sigma_0}) H_0 \Sigma_0 \overline{\Sigma_0} + (\lambda'_2 \Sigma_D^T + \lambda'_3 \overline{\Sigma_D^T}) H_D \Sigma_0 \overline{\Sigma_0}. \quad (36)$$

This has the implication that whenever the RH-triplets in $\mathbf{\Sigma_0} \oplus \overline{\mathbf{\Sigma_0}}$ and the weak doublet in $\mathbf{S_4}$ -singlet $\mathbf{H_0} \subset \mathbf{10_0}$ acquire VEVs v_R and $\alpha_0 \equiv y_0 < H_0 >$, respectively, where the latter is approximately of the order of the weak scale, the weak doublets in $\overline{\mathbf{\Sigma_0}}$ gets an induced VEV. On the otherhand, in addition to v_R , the VEVs of the $\mathbf{S_4}$ -doublet components $\mathbf{H_{1,2}} \subset \mathbf{10_{1,2}}$ generate induced VEVs in the weak doublets in $\overline{\mathbf{\Sigma_{1,2}}}$. The order of magnitudes of all the three induced VEVs can now be expressed as,

$$\langle \Delta_i^\phi \rangle = \frac{v_R^2}{M_U^2} \frac{\alpha_i}{y_i}, (i = 0, 1, 2). \quad (37)$$

where, as defined subsequently in this section, α_i stands for the product of i^{th} VEV and the respective Yukawa coupling y_i .

One major difference from the minimal SUSY $SO(10)$ is that in addition to the VEVs of up- and down type doublets of H_0 , the VEVs of nontrivial $\mathbf{S_4}$ -representations also enter into the RHS of eq.(37).

The next point that needs explanation is how only the six bi-doublets lighter than M_U acquire nearly electroweak-scale VEVs while five of them have masses near M_R scale and only the remaining bi-doublet has mass near the electro-weak scale to supply the up-type and the down-type MSSM doublets (H^u, H^d).

In order to achieve this objective we introduce two $SO(10)$ -singlet scalar fields, η_S and η'_S which transform as doublet and triplet, respectively, under S_4 . These will make additional contribution to the superpotential,

$$W''_H = \lambda_S \eta_S H_0 H_D + \lambda'_S \eta'_S H_0 H_T + \dots \quad (38)$$

We assign order M_R scale VEVs to η_S and η'_S to break the S_4 symmetry at the intermediate scale and generate $H_0 - H_D$ and $H_0 - H_T$ mixings. Then using bi-unitary transformation on the six doublet fields to diagonalize the bi-doublet mass matrix at the intermediate scale, we treat only one linear combinations of the weak bi-doublets to have mass at the electro-weak scale while the remaining five linear combinations are treated to acquire mass at the intermediate scale. This would involve only one fine-tuning of the new parameters of the superpotential. Once the lightest linear combination of all the six bi-doublets is constructed in this manner to supply the MSSM Higgs doublets (H^u, H^d), the electroweak VEVs of the latter imply VEVs of approximately the same order for all the bi-doublet components in $H_i (i = 0, 1, \dots, 5)$. Then the induced VEVs of weak bi-doublets at the GUT-scale contained in $(\Sigma \oplus \overline{\Sigma})_{0,1,2}$, already discussed in eq.(37) follow in a straight-forward manner.

To have a rough idea of the order of the VEVs involved, using eq.(37) and taking the respective Yukawa couplings in the range $y_i \simeq 0.01 - 1.0$ and $\alpha_i \simeq 100$ GeV, $v_R = 10^{13}$ GeV to 10^{14} GeV, $M_U = 2 \times 10^{16}$ GeV, we get $\langle \Delta_i^\phi \rangle = 10$ MeV to 10 GeV. Our numerical analysis approximately agrees with these results.

B. Fermion Masses from S_4 Flavor Symmetry

Investigation on fermion masses and mixings using an $SO(10) \times S_4$ model but without any intermediate gauge symmetry has been carried out in Ref. [40] where RG-extrapolated values of charged fermion masses at the GUT scale have been used to fix certain model parameters and make predictions in the neutrino sector. Although $v_R = 10^{13} - 10^{14}$ GeV has been assumed with a view to obtain the right-handed neutrino mass $M_N = f_0 v_R = 10^{13} - 10^{14}$ GeV for $f_0 \simeq 1$, we note that it is difficult to visualise any such value of v_R substantially lower than the GUT-scale in a single-step breaking scenario; hence the desired value of M_R is not obtainable without adjusting the value of the

Majorana coupling to $f_0 \approx 0.001 - 0.01$. Also we note that the right choice for the input values of charged fermion masses is desirable to be at the intermediate seesaw scale rather than the GUT scale. We carry out investigations utilising the RG-extrapolated values at $M_R \approx v_R \approx 10^{13}$ GeV in the present model where no adjustment of f_0 is needed to obtain the desired see-saw scale. We find that the experimental data on neutrino mass-squared differences and mixings in fact determine the seesaw scale to be $M_N \simeq 3.78 \times 10^{13}$ GeV.

Consistent with the $SO(10) \times S_4$ symmetry the superpotential for fermion-Higgs Yukawa interaction is written as,

$$\begin{aligned} W_{Yuk}^0 = & (\Psi_1 \Psi_1 + \Psi_2 \Psi_2 + \Psi_3 \Psi_3)(Y_0 H_0 + f_0 \bar{\Sigma}_0) + \frac{1}{\sqrt{2}}(\Psi_2 \Psi_2 - \Psi_3 \Psi_3)(y_1 H_1 + f_1 \bar{\Sigma}_1) \\ & + \frac{1}{\sqrt{6}}(-2\Psi_1 \Psi_1 + \Psi_2 \Psi_2 + \Psi_3 \Psi_3)(y_1 H_2 + f_1 \bar{\Sigma}_2) \\ & + y_3[(\Psi_2 \Psi_3 + \Psi_3 \Psi_2)H_3 + (\Psi_1 \Psi_3 + \Psi_3 \Psi_1)H_4 + (\Psi_1 \Psi_2 + \Psi_2 \Psi_1)H_5]. \end{aligned} \quad (39)$$

Following the standard notation with $Q(Q^C)$ and $L(L^C)$ for left(right)-handed quark and lepton doublets in left-right symmetric gauge theory, and denoting H_i^ϕ and $\bar{\Delta}_i^\phi$ as the electroweak bi-doublets in H_i and $\bar{\Sigma}_i$, we now write the Yukawa superpotential consistent with $G_{2213} \times S_4$ just below the GUT-symmetry breaking scale for $\mu \sim M_U$, before the electroweak bi-doublets $\bar{\Delta}_i^\phi$ decouple from the superpotential,

$$\begin{aligned} W_{Yuk} = & \Sigma_{k=1}^3 [Q_k^T \tau_2 (y_0 H_0^\phi + f_0 \bar{\Delta}_0^\phi) Q_k^C + L_k^T \tau_2 (y_0 H_0^\phi - 3f_0 \bar{\Delta}_0^\phi) L_k^C] \\ & + \frac{1}{\sqrt{2}} [Q_2^T \tau_2 (y_1 H_1^\phi + f_1 \bar{\Delta}_1^\phi) Q_2^C - Q_3^T \tau_2 (y_1 H_1^\phi + f_1 \bar{\Delta}_2^\phi) Q_3^C \\ & + L_2^T \tau_2 (y_1 H_1^\phi - 3f_1 \bar{\Delta}_1^\phi) L_2^C - L_3^T \tau_2 (y_1 H_1^\phi - 3f_1 \bar{\Delta}_2^\phi) L_3^C] \\ & + \frac{1}{\sqrt{6}} [-2Q_1^T \tau_2 (y_1 H_2^\phi + f_1 \bar{\Delta}_2^\phi) Q_1^C - 2L_1^T \tau_2 (y_1 H_2^\phi - 3f_1 \bar{\Delta}_2^\phi) L_1^C \\ & + Q_2^T \tau_2 (y_1 H_2^\phi + f_1 \bar{\Delta}_2^\phi) Q_2^C + L_2^T \tau_2 (y_1 H_2^\phi - 3f_1 \bar{\Delta}_2^\phi) L_2^C \\ & + Q_3^T \tau_2 (y_1 H_2^\phi + f_1 \bar{\Delta}_2^\phi) Q_3^C + L_3^T \tau_2 (y_1 H_2^\phi - 3f_1 \bar{\Delta}_2^\phi) L_3^C] \\ & + y_3 [Q_2^T \tau_2 H_3^\phi Q_3^C + Q_3^T \tau_2 H_3^\phi Q_2^C + Q_1^T \tau_2 H_4^\phi Q_3^C + Q_3^T \tau_2 H_4^\phi Q_1^C + Q_1^T \tau_2 H_5^\phi Q_2^C + Q_2^T \tau_2 H_5^\phi Q_1^C \\ & + (Q \rightarrow L)]. \end{aligned} \quad (40)$$

The up and down type electro-weak doublets in the six bi-doublets of $\mathbf{10}$'s $\subset SO(10)$ acquire VEVs $\langle H_i^u \rangle = v_i^u$ and $\langle H_i^d \rangle = v_i^d (i = 0, 1, \dots, 5)$. The electroweak submultiplets in $\bar{\Sigma}_i$ also acquire induced VEVs $\langle \bar{\Delta}_i^u \rangle$ and $\langle \bar{\Delta}_i^d \rangle$, ($i = 0, 1, 2$).

Adding their contributions, the mass matrices of quarks and leptons have the well known forms,

$$M_u = M_u^{(10)} + M_u^{(126)}, \quad M_d = M_d^{(10)} + M_d^{(126)}, \quad (41)$$

$$M_l = M_d^{(10)} - 3M_d^{(126)}, \quad M_\nu^D = M_u^{(10)} - 3M_u^{(126)}, \quad (42)$$

$$M_\nu = M_\nu^{DT} M_\nu^D / M_N, \quad (43)$$

where $M_N = f_0 v_R$ is the degenerate right-handed neutrino mass and we have already noted in Sec.3 that we can have $M_N \simeq M_R$ substantially below the GUT-scale in this model without having the necessity to adjust the the Majorana coupling to be a small fraction of unity. The component mass-matrix elements in the above equations are defined as,

$$M_u^{(10)} = \begin{bmatrix} \alpha_0 - 2\alpha_2 & \alpha_5 & \alpha_4 \\ \alpha_5 & \alpha_0 + \alpha_1 + \alpha_2 & \alpha_3 \\ \alpha_4 & \alpha_3 & \alpha_0 - \alpha_1 + \alpha_2 \end{bmatrix}, \quad (44)$$

$$M_d^{(10)} = \begin{bmatrix} \beta_0 - 2\beta_2 & \beta_5 & \beta_4 \\ \beta_5 & \beta_0 + \beta_1 + \beta_2 & \beta_3 \\ \beta_4 & \beta_3 & \beta_0 - \beta_1 + \beta_2 \end{bmatrix}, \quad (45)$$

TABLE II: Renormalisation Group extrapolated running masses of quarks and charged leptons of three generations at the intermediate scale $M_R = 10^{13}$ GeV as estimated in Ref. [41].

$\tan \beta$	10	55
m_u (MeV)	$0.888 \pm_{0.179}^{0.169}$	$0.888 \pm_{0.179}^{0.167}$
m_c (MeV)	$258.094 \pm_{25.833}^{23.828}$	$258.292 \pm_{25.814}^{23.329}$
m_t (GeV)	$94.369 \pm_{25.833}^{22.557}$	$104.236 \pm_{18.202}^{32.701}$
m_d (MeV)	$1.829 \pm_{0.277}^{0.511}$	$1.821 \pm_{0.275}^{0.505}$
m_s (MeV)	$36.426 \pm_{5.480}^{5.158}$	$36.289 \pm_{5.434}^{5.077}$
m_b (GeV)	$1.263 \pm_{0.089}^{0.118}$	$1.576 \pm_{0.168}^{0.264}$
m_e (MeV)	$0.391 \pm_{0.0002}^{0.0002}$	$0.389 \pm_{0.0002}^{0.0005}$
m_μ (MeV)	$82.553 \pm_{0.033}^{0.034}$	$82.206 \pm_{0.102}^{0.046}$
m_τ (GeV)	$1.408 \pm_{0.0008}^{0.0009}$	$1.657 \pm_{0.014}^{0.018}$

$$M_u^{(126)} = \begin{bmatrix} \gamma_0 - 2\gamma_2 & 0 & 0 \\ 0 & \gamma_0 + \gamma_1 + \gamma_2 & 0 \\ 0 & 0 & \gamma_0 - \gamma_1 + \gamma_2 \end{bmatrix}, \quad (46)$$

$$M_d^{(126)} = \begin{bmatrix} \delta_0 - 2\delta_2 & 0 & 0 \\ 0 & \delta_0 + \delta_1 + \delta_2 & 0 \\ 0 & 0 & \delta_0 - \delta_1 + \delta_2 \end{bmatrix}. \quad (47)$$

In these equations $\alpha_i \equiv y_i < H_i^u >$, $\beta_i \equiv y_i < H_i^d >$, $\gamma_i \equiv f_i < \Delta_i^u >$ and $\delta_i \equiv f_i < \Delta_i^d >$ (i not summed). The choice of diagonal basis in the down quark sector which automatically also leads to the diagonal basis in the charged lepton sector, enables to choose the six parameters, $\beta_i, \delta_i (i = 0, 1, 2)$ to be real and $\beta_3 = \beta_4 = \beta_5 = 0$. All other parameters are, in general, complex. Analytically we express the six real parameters in terms of down-quark and charged lepton mass eigen-values at the see-saw scale ($\mu = M_R$),

$$\begin{aligned} \beta_0 &= [3(m_b^0 + m_s^0 + m_d^0) + m_\tau^0 + m_\mu^0 + m_e^0] / 12, \\ \beta_1 &= [-3m_b^0 + 3m_s^0 - m_\tau^0 + m_\mu^0] / 8, \\ \beta_2 &= [3m_b^0 + 3m_s^0 - 6m_d^0 + m_\tau^0 + m_\mu^0 - 2m_e^0] / 24, \\ \delta_0 &= [m_b^0 + m_s^0 + m_d^0 - (m_\tau^0 + m_\mu^0 + m_e^0)] / 12, \\ \delta_1 &= [-m_b^0 + m_s^0 + m_\tau^0 - m_\mu^0] / 8, \\ \delta_2 &= [m_b^0 + m_s^0 - 2m_d^0 - m_\tau^0 - m_\mu^0 + 2m_e^0] / 24. \end{aligned} \quad (48)$$

We utilise the RG-extrapolated values of the running charged fermion masses at the intermediate scale $\mu = M_R \approx v_R \approx 10^{13}$ GeV as shown in Table.II for $\tan \beta = 10, 55$ [41]. In the present model the definition $\tan \beta = v_u/v_d$ is valid in the presence of MSSM below the intermediate scale.

Using the down quark and charged lepton masses from Table II and eqs. (48) we obtain,

$$\beta_0 = 449.773 \text{ MeV}, \beta_1 = -625.971 \text{ MeV}, \beta_2 = 224.155 \text{ MeV}, \quad (49)$$

$$\delta_0 = -15.791 \text{ MeV}, \delta_1 = 12.334 \text{ MeV}, \delta_2 = -8.074 \text{ MeV}. \quad (50)$$

Using low-energy values of CKM matrix elements with its phase $\delta = 60^\circ$ and using the renormalization factor $r_N = \exp[-(y_{top}^2 \ln(v_R/m_{top})/16\pi^2)] \simeq 0.86$ leads to the CKM matrix at $\mu = M_R = 10^{13}$ GeV,

$$V_{CKM} = \begin{bmatrix} 0.973852 & 0.22720 & 0.00169097 - 0.00292880i \\ -0.227985 - 0.000134610i & 0.97301 - 0.000031405i & 0.0369880 \\ 0.00675842 - 0.002851022i & -0.0364054 - 0.000664840i & 0.99925 \end{bmatrix}. \quad (51)$$

Defining $\hat{M}_u = \text{diag}(m_u^0, m_c^0, m_t^0)$, at first we obtain elements of M_u in terms of the running up-quark masses and CKM elements via,

$$M_u = V_{CKM}^T \hat{M}_u V_{CKM}. \quad (52)$$

For $\tan \beta = 10$, using eqs.(44), (46), (51), and (52) and Table II, determines the three parameters $\alpha_i (i = 3, 4, 5)$ while three equations are obtained among the other six complex parameters, $\alpha_i (i = 0, 1, 2)$ and $\gamma_i (i = 0, 1, 2)$,

$$\begin{aligned} \alpha_0 + \alpha_1 + \alpha_2 + \gamma_0 + \gamma_1 + \gamma_2 &= 369.414 \pm_{42.847}^{53.550} - i(4.583 \pm_{0.702}^{1.099}), \\ \alpha_0 - \alpha_1 + \alpha_2 + \gamma_0 - \gamma_1 + \gamma_2 &= 94204.062 \pm_{14450.556}^{22517.333} - i8.797 \times 10^{-6}, \\ \alpha_0 - 2\alpha_2 + \gamma_0 - 2\gamma_2 &= 17.687 \pm_{1.832}^{3.085} - i(3.6192 \pm_{0.556}^{0.867}), \end{aligned} \quad (53)$$

$$\begin{aligned} \alpha_3 &= -(3423.163 \pm_{525.630}^{756.854}) - i(62.685 \pm_{9.616}^{14.983}), \\ \alpha_4 &= 634.282 \pm_{96.749}^{152.893} - i(268.690 \pm_{41.215}^{64.225}), \\ \alpha_5 &= -(80.018 \pm_{9.056}^{11.102}) + i(9.334 \pm_{1.434}^{2.185}), \end{aligned} \quad (54)$$

where all parameters are in MeV and the uncertainties in the RHS of these equations reflect the uncertainties in the low-energy data [41]. It is clear that the set of three eqs.(53) leaves undetermined three complex (six real) parameters which provide a very rich structure to the model. Because of this, the model may be able to confront the present neutrino data and even the future precision data that may emerge from planned and ongoing oscillation experiments. On the other hand, it is also possible that the number of parameters may not ensure faithful representation of neutrino data because of highly non-linear nature of the problem emerging from see-saw mechanism.

In order to examine the efficiency of the model in representing the neutrino sector, we use the standard parametrization of the leptonic Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix,

$$U_{PMNS} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \text{diag}(e^{-i\varphi_1/2}, e^{-i\varphi_2/2}, 1), \quad (55)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, δ is the Dirac phase and φ_1, φ_2 are Majorana phases of neutrinos. These phases have range from 0 to 2π .

We use experimental data on neutrino oscillations within the 3σ limit [42]:

$$\begin{aligned} 0.29 &\leq \tan^2 \theta_{12} \leq 0.64, \\ 0.49 &\leq \tan^2 \theta_{23} \leq 2.2, \\ \sin^2 \theta_{13} &\leq 0.054, \\ 5.2 &\leq \Delta m_{\odot}^2 / 10^{-5} eV^2 \leq 9.8, \\ 1.4 &\leq \Delta m_{atm}^2 / 10^{-3} eV^2 \leq 3.4. \end{aligned} \quad (56)$$

For numerical analysis we exploit the well defined diagonalisation procedure for complex and symmetric mass matrices,

$$\begin{aligned} U^\dagger M_\nu U^* &= \text{diag}(m_1, m_2, m_3), \\ U^\dagger M_\nu M_\nu^\dagger U &= \text{diag}(m_1^2, m_2^2, m_3^2), \end{aligned} \quad (57)$$

where U is a unitary diagonalising matrix, the light neutrino mass matrix M_ν has been defined in eq.(43) and $m_i (i = 1, 2, 3)$ are positive mass eigen values.

For the sake of simplicity we reduce the parameters of the model by treating the parameters $\gamma_i (i = 0, 1, 2)$ as real. Then eq. (53) determines six real parameters out of a total nine, including real and imaginary parts of $\alpha_i (i = 0, 1, 2)$

TABLE III: Fit to the available neutrino oscillation data and predictions of reactor mixing angle θ_{13} , leptonic Dirac phase (δ), Majorana phases (φ_1, φ_2) and the CP violation parameter J_{CP} in the $SO(10) \times S_4$ model with see-saw scale at $M_R = 3.78 \times 10^{13}$ GeV and $\tan \beta = 10$

ξ (GeV)	1.025	1.100	1.235
η (GeV)	2.137	2.137	2.400
ζ (GeV)	25.529	25.529	25.700
m_1 (eV)	0.00536	0.00596	0.00801
m_2 (eV)	0.00920	0.00956	0.01268
m_3 (eV)	0.05000	0.05000	0.07860
$\sum_i m_i$ (eV)	0.0645	0.0675	0.09929
Δm_{\odot}^2 (eV ²)	6×10^{-5}	6×10^{-5}	9.6×10^{-5}
Δm_{atm}^2 (eV ²)	2.5×10^{-3}	2.5×10^{-3}	3.1×10^{-3}
$\sin \theta_{12}$	0.515	0.616	0.511
$\sin \theta_{23}$	0.718	0.718	0.736
$\sin \theta_{13}$	0.055	0.057	0.052
δ (radians)	3.096	3.048	3.100
ϕ_1 (radians)	5.67	5.46	5.65
ϕ_2 (radians)	5.59	5.39	5.65
J_{CP}	2.66×10^{-4}	6.49×10^{-4}	2.95×10^{-5}
$< m_{ee} >$ (eV)	0.00646	0.00742	0.00932
m_{β} (eV)	0.00462	0.00516	0.00600

and real γ_i ($i = 0, 1, 2$). This choice of parameters implies that the CP-violation has its origin only in the quark sector as reflected in the CKM matrix [43].

Thus, in addition to the see-saw scale, we are left with three real parameters to fit the neutrino oscillation data on four quantities, Δm_{\odot}^2 , Δm_{atm}^2 , $\tan^2 \theta_{12}$, and $\tan^2 \theta_{23}$ and make predictions on $\sin \theta_{13}$, leptonic Dirac phase (δ) and Majorana phases (φ_1, φ_2), sum of the three light neutrino masses Σm_i , the effective matrix element for neutrinoless double beta decay, $< m_{ee} >$, and the kinematic neutrino mass m_{β} to be measured in beta decay where

$$< m_{ee} > = |\sum_{i=1}^3 (U_{PMNS}^{ei})^2 m_i|, \quad m_{\beta} = (\sum_{i=1}^3 |U_{PMNS}^{ei}|^2 m_i^2)^{1/2}. \quad (58)$$

Equivalently, the three real unknown parameters are defined as,

$$\begin{aligned} \xi &= \gamma_0 - 2\gamma_2, \\ \eta &= \gamma_0 + \gamma_1 + \gamma_2, \\ \zeta &= \gamma_0 - \gamma_1 + \gamma_2. \end{aligned} \quad (59)$$

Even in the constrained parametrisation of the model, we find that ξ , η and ζ are quite efficient in describing the present neutrino oscillation data. Some examples of our fit to the data and model predictions are shown in Table III.

We find that the see-saw scale is determined to be $M_N = 3.78 \times 10^{13}$ GeV for hierarchial neutrino masses. The first and the second columns show that for fixed values of η and ζ , the parameter ξ is very effective in controlling the value of the solar neutrino mixing angle (θ_{12}). Within the uncertainties shown in eq.(53) and eq.(54), the predicted reactor mixing angle occurs in the range $\theta_{13} \simeq 3^\circ - 5^\circ$ which is within the accessible limit of ongoing and planned experiments [44]. The sum of the three neutrino masses are found to be well within the cosmological bound [45]. The leptonic Dirac phase turns out to be closer to π with $\delta = 2.9 - 3.1$ radians and the two Majorana phases are within $5.3 - 5.7$ radians. The predicted values of matrix element for double beta decay and the kinematical mass for beta decay are found to be nearly two orders smaller than the current experimental bounds [46, 47, 48]. Similar conclusion has been also obtained for hierarchial neutrinos with S_4 flavor symmetry in the non-SUSY standard model [28]. The Jarlskog invariant [49] is found to vary between $J_{CP} \simeq 2.95 \times 10^{-5}$ and $J_{CP} \simeq 10^{-3}$ where the smaller (larger) value depends upon how much closer (farther) is the Dirac phase (δ) from π . We observe that the predictions of this model in the neutrino sector made at the high see-saw scale is to remain stable under radiative corrections when extrapolated to low energies especially since the light neutrino mass eigen values are small [50].

VII. SUMMARY AND CONCLUSION

In this work we have addressed the question of possible existence of R-parity and Parity conserving left-right gauge theory as an intermediate symmetry in supersymmetric $SO(10)$ grand unified theory with manifest one-loop unification of the gauge couplings. We found that it is possible to have this intermediate gauge symmetry provided both the left-right gauge theory and $SO(10)$ are extended to contain S_4 flavor symmetry. The particle spectrum needed to implement the gauge coupling unification is found to match into different $G_{2213} \times S_4$ representations leading to exactly vanishing values of two RG coefficients. The Higgs spectrum is also found to be consistent with the mass spectra analysis for the $SO(10) \times S_4$ model with fine-tuning of certain parameters in the Higgs superpotential. At first ignoring the light neutrino mass constraint, the left-right symmetry breaking scale is allowed to have a wide range of values with $M_R = 5 \times 10^9$ GeV to 10^{15} GeV and no tuning of the Majorana coupling is needed in this model to obtain desired value of the see-saw scale substantially below the GUT scale.

We have carried out analysis of fermion masses and mixings using RG extrapolated values of the low-energy data at the intermediate scale $M_R \simeq 10^{13}$ GeV in SUSY $SO(10) \times S_4$ for the first time. Even in the case of constrained model parametrisation where CP-violation originates only from the quark sector through CKM matrix, the model is found to fit all values of quark and lepton masses and mixings including very large values of mixings and very small values of masses in the neutrino sector. The neutrino oscillation data determines the see-saw scale to be $M_N \simeq 3.8 \times 10^{13}$ GeV for hierarchical neutrinos. Apart from predictions on leptonic CP-violating parameter, Dirac and Majorana phases, the predicted values of the reactor-neutrino mixing angle, $\theta_{13} \simeq 3^\circ - 5^\circ$, are accessible to ongoing and planned long baseline experiments on neutrino oscillations.

It would be interesting to investigate prospects of this model with all complex parameters in type I see-saw and the case of experimentally testable quasi-degenerate neutrino spectrum with type II see-saw or a combination of both the type I and type II see-saw models in future works.

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